

On the analytical representation of the characteristic curve of photographic emulsions

ESCHRICH, K.-O.

Zentralinstitut für Astrophysik der Akademie der Wissenschaften der DDR, Potsdam-Babelsberg (DDR)

Summary

Summarising the kinetic theory of the photographic process simple formulae of the characteristic curve are derived, making use of special approximations that allow for elementary integration. The influence of the distinguished parameters on formation of the characteristic curve is investigated.

Резюме

Обобщающая кинетическая теория фотографического процесса. Выведена упрощенная формула характеристической кривой, используя специальную аппроксимацию, которая позволяет элементарное интегрирование. Исследовано влияние некоторых параметров на формирование характеристической кривой.

Zusammenfassung

Die kinetische Theorie des photographischen Prozesses wird kurzgefaßt dargelegt. Die spezielle Wahl von Funktionen, die die Approximation realer Zusammenhänge sind, erlaubt elementare Integrationen mit dem Ergebnis analytischer Darstellungen der Schwärzungskurven. Anhand dieser Formeln wird der Einfluß verschiedener Parameter auf die Form der Schwärzungskurve untersucht.

In astrophysics, as commonly in many branches of experimental natural science, photography is important for reception, storage, and transmission of information.

Although there have been appeared other modern means of detections radiation, it should be emphasized that the photoemulsion is a suitable detector for electromagnetic radiation in the optical region, possessing both an incomparably high storage density of information and a relatively good resolving power. Moreover, in the archives of the observatories lots of photographic plates are stored, having to be reduced with the knowledge of the relation between exposure and photographic density. One should emphasize that the photographic process is one of the most complicated one although the photographic layer may be regarded as one of the oldest artificial radiation detectors.

The reception of the information in the photographic layer by the photoelectrical effect is followed by a series of consecutive photochemical processes. This leads to a montone

functional relation between the photographic density and exposure. We base our treatment mainly on the theories of GURNEY, MOTT and MITCHELL, as described in detail in the monography by MEES [1]. We refer to several publications by GERTH (for referencis see below), where the kinetics of the speck transformation and the density function have been treated.

In a following paper, the inverse function will be derived under certain restriction as they result from astrophysical tasks.

1. Principal relations in the theory of the photographic process

In this chapter a short treatment of the kinetic theory of the photographic process [2] is given, which serves as the basis for a derivation of the analytic function of the characteristic curve.

The photographic layer is an emulsion of light-sensitive grains of silver halide and gelatine. At first we consider a single silver halide grain. By action of the internal photoelectric effect in the crystalline lattice N_e free electrons (concentration c_e) and N_d electron holes (concentration c_d) are produced. This process may be described (simplified) by the coupled nonlinear differential equations

$$\frac{dN_e}{dt} = \eta E - \alpha_e \cdot N_e - \beta \cdot N_e \cdot N_d, \quad (1)$$

$$\frac{dN_d}{dt} = \eta E - \alpha_d \cdot N_d - \beta \cdot N_e \cdot N_d, \quad (2)$$

where η and E are the sensitivity coefficient of the photoelectric effect, and the intensity of the incident light respectively. Further α_e denotes the coefficient of the electron traps, α_d denotes the traps of the holes, and β is the coefficient describing the recombination of the electrons and holes.

Starting from an initial state with $N_e(0) = N_d(0) = 0$ a linear increase with time t occurs. After elapsing of a sufficient exposure time ($t > 10^{-3}$ s) a state of equilibrium is reached, which is formulated by the equations

$$N_e(\infty) = \frac{\alpha_d}{2\beta} \left(\sqrt{1 + \frac{4\beta}{\alpha_e \cdot \alpha_d} \eta E} - 1 \right), \quad N_d(\infty) = \frac{\alpha_e}{2\beta} \left(\sqrt{1 + \frac{4\beta}{\alpha_e \cdot \alpha_d} \eta E} - 1 \right). \quad (3)$$

Caused by the charge action of the free electrons elementary silver atoms arise at sensitivity specks of the crystal lattice representing special steps in the sequence of growing specks, constituting a kinetic MARKOVIAN chain. The specks in the individual kinetic steps have different physical and chemical properties, especially only specks with 4 and more atoms are developable (an aggregate of 4 atoms in a spatial arrangement represents a microcrystal of metallic silver). Different specks generated by a multi-step kinetic reaction are contained in an exposed silver halid crystal.

The concentration of a definite specimen of a speck increases by accretion from the neighbouring lower and upper steps whereas it decreases by dilution. The coefficients μ_i of the build-up reactions (in direction from the lower to the upper steps) are proportional to the electron concentration, $\mu_i = \mu_i^* \cdot N_e$. The back reactions are caused by thermal decay; the coefficients of these reactions are denoted by ν_i . So we have the balance

equation for the i -th speck-step

$$\frac{dc_i}{dt} = \mu_i \cdot c_{i-1} - (\mu_{i+1} + \nu_i) \cdot c_i + \nu_{i+1} \cdot c_{i+1}, \quad i = 0, 1, \dots, M, \quad (4)$$

or

$$\frac{dc_i}{dt} = K_{ij} \cdot c_j, \quad i = 0, 1, \dots, M, \quad (5a)$$

with

$$K_{ii} = -(\mu_{i+1} + \nu_i), \quad K_{ii-1} = \mu_i, \quad K_{ii+1} = \nu_{i+1}, \quad (5b)$$

all other, $K_{ij} = 0$ (tridiagonal-matrix). The lowest speck-step is designate with 0 ($i = 0$), so that $c_i = 0$ yields for $i < 0$. Thus we have $\mu_0 = \nu_0 = 0$, i.e., for c_0 there is no growth from lower steps and no reduction to lower steps.

The solution of the equation (4 or 5) with the initial conditions $c_i(0)$ at the time $t = 0$ is given by

$$c_i(t) = B_{ij}(E, t) \cdot c_j(0), \quad i, j = 0, 1, \dots, M. \quad (6)$$

As the action of the exposure the kinetic balance is shifted causing a redistribution of the speck concentrations related to the steps. The analytic description of this process is, that the initial vector $c_j(0)$ is transformed into the vector of the final distribution of concentrations $c_j(t)$ described by the exposure matrix B_{ij} , depending on the time t and the intensity function $E(\tau)$ with $0 \leq \tau \leq t$. In the special case $E = \text{const}$ the functional representation of the transformation matrix may be written in the form

$$B_{ij} = e^{K_{ij}t}. \quad (7)$$

Denoting the coefficient of developability of the i -th speck-step w_i , ($w_{i < 4} = 0$, $w_{i \geq 4} > 0$), we have the concentration of developable specks as the vector product $w_i \cdot c_i(t)$. The number of developable specks a grain is

$$z = V \cdot w_i \cdot c_i(t) = V \cdot w_i \cdot B_{ij} \cdot c_j(0), \quad (8)$$

wherin V is the light sensitive part of the grain volume. The probability of development of a single grain is given after SVEDBERG (based on the POISSON law of distribution) by

$$w = 1 - e^{-z}, \quad (9)$$

which yields for the density of a thin emulsion layer with grains of uniform size and the mean value of developable specks \bar{z}

$$S = S_\infty \cdot (1 - e^{-\bar{z}}). \quad (10)$$

S denoting the density and S_∞ the saturation density of a thin layer. The light sensitive volume V of a grain can be represented as a power function

$$V = a^{3\kappa} \quad (11)$$

with a being a characteristic length of the grains, and the exponent κ takes into account the shape of the grain and the absorption of the radiation. For $\kappa = 0$ the sensitivity is independent of the size of the grains (as in case of physical development), for $\kappa = 2/3$

the sensitivity depends on the surface area of the grain, and for $\varkappa = 1$ we have a proportionality of the sensitivity to the volume of the grain. Commonly \varkappa takes values between 0 and 1. Taking into account the normalized distribution function of the grain sizes $\omega(a)$ we have to integrate the densities for all classes of grains over a . The contribution of one grain to the density is proportional to the projected area, i.e., to the square of the characteristic length. The mean projected area \bar{F} is

$$\bar{F} \sim \bar{a}^2 = \int_0^{\infty} a^2 \cdot \omega(a) \cdot da, \quad (12)$$

which serves for the normalization. A further integration has to be performed over the depth x of the layer with the total thickness x_0 . Because of the opacity D of the emulsion layer and reflections at the boundaries the effective intensity is depthdependent, we have

$$E_{\text{eff}}(x) = E \cdot \varphi(x). \quad (13)$$

Both integrations together yield the basic formula of the characteristic curve, containing all influences of exposure, grain distribution, and transfer of radiation within the layer

$$S = \frac{S_{\infty}}{a^2 \cdot x_0} \int_0^{x_0} \int_0^{\infty} a^2 \cdot \omega(a) \cdot [1 - e^{-V(a) \cdot w_1 \cdot B_{1j}(E \cdot \varphi(x), t) \cdot c_j(0)}] \cdot da \, dx. \quad (14)$$

2. Conclusions from the theory of the photographic process

2.1. Schwarzschild's law

Any theory of the photographic process has to be checked by the wellknown law of blackening found by SCHWARZSCHILD, which can be regarded as the most critical criterion of its truth, as shown by GERTH [3] in detail. Here we restrict ourselves to the results.

SCHWARZSCHILD'S law

$$E \cdot t^p = \text{const} \quad (15)$$

describes for a definite constant and likewise for a definite density S a relation between the intensity E of the incident light and the exposure time t . p is the SCHWARZSCHILD exponent ranging in $0 < p \leq 1$ for long-time exposures (such as for astronomical purposes). Hence the resultant density is not determined by the exposure energy (per unit area) $H = E \cdot t$, corresponding to the reciprocity law, but by some time-dependent process, which diminishes the efficiency of the exposure with progress in time using $H = E \cdot t^p$ as the independent variable of the exposure-density function.

To demonstrate this assertion we go out of the basic formula for the density function equation (14). All properties of the exposure related to the intensity and time are contained in the exponent of the integrand, so we can confine in the following consideration to this exponent, that means, to the mean number of developable specks on a grain. Therefore, for constant density follows

$$w_1 \cdot B_{1j}(E \cdot \varphi(x), t) \cdot c_j(0) = \text{const} \quad (16)$$

which is termed as the universal exposure relation.

In order to derive SCHWARZSCHILD's law out of equation (16) we restrict ourselves to the consideration of the very simple case, that in the emulsion before exposure only specks of the order $i = 0$ are contained; i.e., $c_0 \neq 0, c_i = 0, \dots$ (unsensitized emulsion — or any single uniform step $i \leq 3$). Moreover, only specks of the order $i = n$ should be developable, i.e., $w_n = 1, w_{i \neq n} = 0$. Then, equation (16) is reduced to the scalar relation

$$B_{n0}(E_{\text{eff}}, t) = \text{const.} \tag{17}$$

By means of further simplified but physically founded assumptions [3] one gets the approximated value of B_{n0}

$$B_{n0}(E, t) = \sigma_1^n \cdot N_e^n(E) \cdot t^{n-m} = (\sigma_1 \cdot N_e(E) \cdot t^p)^n = \text{const}, \quad p = \frac{n - m}{n}. \tag{18}$$

Using equation (3) and inserting the abbreviation $a = 4\beta\eta/\alpha_e\alpha_d$,

$$\left[\sqrt{1 + aE} - 1\right] \cdot t^p)^n = \text{const} \tag{19}$$

results. Herein σ_1 is a function of other constants, and m is the number of steps with back reactions ($v_i \neq 0$). If $m = 0$, then $p = 1$, i.e., any reciprocity failure does not occur. The highest value of m is $m = n$ yielding $p = 0$. This is the case for extreme low exposure intensities, the density being independent of the exposure time, namely zero. As mentioned above, the number of atoms contained in a developable speck must be at least 4, since only a spatial arrangement of atoms constitutes microcrystals capable to disturb the surface potential of the grain. In the case of a nonsensitized emulsion there are present on the grain only specks of the order zero; i.e., the surface of the crystal contains no free silver atoms. Therefore, to reach developability the specks have to transit 4 reaction steps. In the other case of a high sensitized emulsion the grains are occupied by specks of the order 3, i.e., the surface of the crystal contains coagulations of 3 silver atoms constituting still no microcrystalline metallic silver and being not developable. Since in real emulsions specks of different order exist, the magnitudes m and n represent mean values from integers corresponding to the single kinetic steps. So we have

$$0 < p \leq 1, \quad \text{since} \quad 0 \leq m < n. \tag{20}$$

Consequently, the value n is near unity for high sensitive emulsions, but near 4 for emulsions of low sensitivity [4]. The exponent n is the universal effective step number, introduced by E. GERTH, representing the reaction order of a multistep kinetic reaction [5]. For low intensities with $E \ll 1/a$ we obtain from equation (19) SCHWARZSCHILD's law (15). This intensities govern the so called *Schwarzschild domain* of exposure. On the other hand, a SCHWARZSCHILD law comes out for high intensities with $E \gg 1/a$ as applied at short-time exposures, with an SCHWARZSCHILD exponent $p^* = 2 \cdot p$. By this way values of the SCHWARZSCHILD exponent $p^* > 1$ as found by KRON [6, 7] become comprehensible.

In astronomic photography the exposure intensities are situated in the *Schwarzschild domain*, so that for the exposure matrix yields

$$B_{n0}(E, t) = B_{n0}([E \cdot t^p]^n) = \eta \cdot (E \cdot t^p)^n \tag{21}$$

(properly speaking $B_{n3}(E, t)$) with the effective exposure energy

$$H = E \cdot t^p \tag{22}$$

and

$$\varepsilon = W_n \cdot \eta \cdot c_0(0) \tag{23}$$

one obtains as the analytical representation of the characteristic curve

$$S(H) = S_\infty \cdot \left[1 - \frac{1}{a^2 \cdot x_0} \int_0^{x_0} \int_0^\infty a^2 \cdot \omega(a) \cdot e^{-V(a) \cdot \varepsilon \cdot H^n \cdot \varphi^n(x)} \cdot da \cdot dx \right]. \tag{24}$$

2.2. Characteristic functions derived under special assumptions

Explicite characteristic functions should be derived from equation (24). In this case some properties of photographic emulsions, as for instance solarisation, are excluded. We are interested in the application of photography in astrophysics rendering the derivation given here sufficient. The fog should be taken into account by an additive term in the exponent of the equation (24). This again gives an additive term of the density itself, which can be determined seperately. Therefore, to evaluate the integral over a , the functions of the grain-size distribution $\omega(a)$ and of the sensitive volume are required. The grain-size follows a lognormal distribution as found experimentally. A suitable approximation of this distribution function is given by the difference of two exponential functions,

$$\omega(a) = A \cdot (e^{-\alpha_1 \cdot a^\lambda} - e^{-\alpha_2 \cdot a^\lambda}), \quad 1 \leq \lambda \leq 3, \quad \alpha_2 > \alpha_1, \tag{25}$$

possessing a nearly bell-shaped curvature for the parameters $\lambda = 3$, $\alpha_1 = 1$, $\alpha_2 = 16$ related to the coordinate $\ln a$; A being a normalizing factor.

If we assume, that

$$V(a) = a^\lambda, \quad (\text{i.e., } \lambda = 3x), \tag{26}$$

is in relatively good agreement with the reality, then the integration of equation (24) over a proves to be very simple. We obtain

$$S(H) = S_\infty \left[1 - \frac{1}{x_0} \cdot \int_0^{x_0} \frac{dx}{(1 + \varepsilon \cdot (H \cdot \varphi)^n / \alpha_1) \cdot (1 + \varepsilon \cdot (H \cdot \varphi)^n / \alpha_2)} \right], \quad \lambda = 3, \tag{27}$$

$$S(H) = S_\infty \left[1 - \frac{1}{x_0} \cdot \frac{1}{\alpha_2^{3/2} - \alpha_1^{3/2}} \cdot \int_0^\infty \frac{\alpha_2^{3/2} \cdot (1 + \varepsilon \cdot (H \cdot \varphi)^n / \alpha_2)^{3/2} - \alpha_1^{3/2} \cdot (1 + \varepsilon \cdot (H \cdot \varphi)^n / \alpha_1)^{3/2}}{(1 + \varepsilon \cdot (H \cdot \varphi)^n / \alpha_2)^{3/2} \cdot (1 + \varepsilon \cdot (H \cdot \varphi)^n / \alpha_1)^{3/2}} \cdot dx \right], \quad \lambda = 2. \tag{28}$$

These two functions for $\varphi(x) \equiv 1$, neglecting the opacity of the emulsion, would yield the characteristic functions

$$S(H) = S_\infty \left[1 - \frac{1}{(1 + \varepsilon \cdot H^n/\alpha_1) \cdot (1 + \varepsilon \cdot H^n/\alpha_2)} \right], \quad \lambda = 3 \tag{29}$$

and

$$S(H) = S_\infty \left[1 - \frac{1}{\alpha_2^{3/2} - \alpha_1^{3/2}} \cdot \frac{\alpha_2^{3/2} \cdot (1 + \varepsilon \cdot H^n/\alpha_2)^{3/2} - \alpha_1^{3/2} \cdot (1 + \varepsilon \cdot H^n/\alpha_1)^{3/2}}{\alpha_2^{3/2} \cdot (1 + \varepsilon \cdot H^n/\alpha_2)^{3/2} \cdot \alpha_1^{3/2} (1 + \varepsilon \cdot H^n/\alpha_1)^{3/2}} \right], \tag{30}$$

$$\lambda = 2.$$

If the extinction within the emulsion layer follows LAMBERT's law

$$\varphi(x) = e^{-D \cdot x/x_0}, \tag{31}$$

then from (27) the characteristic function

$$S(H) = \frac{S_\infty}{2nD} \left\{ \ln \frac{(1 + \varepsilon \cdot H^n/\alpha_1) \cdot (1 + \varepsilon \cdot H^n/\alpha_2)}{(1 + \varepsilon \cdot H^n e^{-nD}/\alpha_1) \cdot (1 + \varepsilon \cdot H^n e^{-nD}/\alpha_2)} + \frac{\alpha_2 + \alpha_1}{\alpha_2 - \alpha_1} \cdot \ln \frac{(1 + \varepsilon \cdot H^n/\alpha_1) \cdot (1 + \varepsilon \cdot H^n \cdot e^{-nD}/\alpha_2)}{(1 + \varepsilon \cdot H^n \cdot e^{-nD}/\alpha_1) (1 + \varepsilon \cdot H^n/\alpha_2)} \right\}, \quad \lambda = 3, \tag{32}$$

can be drawn. On the other hand, starting from an incorrect but the results only weekly influencing grain-size distribution,

$$\omega(a) = A \cdot e^{-x_1 \cdot a^2}, \tag{33}$$

which is obtained from (25) by $\alpha_2 \rightarrow \infty$, we get the following simplified relations:

$$S(H) = S_\infty \left(1 - \frac{1}{1 + \varepsilon \cdot H^n/\alpha_1} \right), \quad \lambda = 3, \quad \varphi \equiv 1, \tag{34}$$

$$S(H) = S_\infty \left(1 - \frac{1}{(1 + \varepsilon \cdot H^n/\alpha_1)^{3/2}} \right), \quad \lambda = 2, \quad \varphi \equiv 1, \tag{35}$$

$$S(H) = \frac{S_\infty}{nD} \ln \frac{(1 + \varepsilon \cdot H^n/\alpha_1)}{(1 + \varepsilon \cdot H^n \cdot e^{-nD}/\alpha_1)}, \quad \lambda = 3. \tag{36}$$

The integration of (28) inserting the functions (31) and (33) yields

$$S(H) = S_\infty \left\{ 1 - \frac{1}{nD} \left[\frac{2}{(1 + \varepsilon \cdot H^n/\alpha_1)^{1/2}} - \frac{2}{(1 + \varepsilon \cdot H^n \cdot e^{-nD}/\alpha_1)^{1/2}} + \ln \frac{((1 + \varepsilon \cdot H^n \cdot e^{-nD}/\alpha_1)^{1/2} + 1) \cdot ((1 + \varepsilon \cdot H^n/\alpha_1)^{1/2} - 1)}{((1 + \varepsilon \cdot H^n/\alpha_1)^{1/2} + 1) \cdot ((1 + \varepsilon \cdot H^n \cdot e^{-nD}/\alpha_1)^{1/2} - 1)} \right] \right\}, \quad \lambda = 2. \tag{37}$$

The quantitative differences among the here presented characteristic curves (29), (30), (32) and (34), ..., (37) are insignificant.

2.3. General properties of the characteristic functions

Previously, from the relation (24) the principle properties, e.g., the limiting cases of the characteristic function, could be demonstrated:

$$S(H) \approx S_\infty, \quad \text{for} \quad H \gg (V \cdot \varepsilon \cdot \varphi)^{-1} \tag{38}$$

and

$$S(H) \approx S_\infty \cdot a_1 \cdot H^n, \quad \text{for } H \ll (V \cdot \varepsilon \cdot \varphi)^{-1}. \quad (39)$$

Herein the constant is

$$a_1 = \frac{\varepsilon}{a^2} \int_0^\infty a^2 \cdot \omega(a) \cdot V(a) \cdot da \cdot \frac{1}{x_0} \int_0^{x_0} \varphi^n(x) dx. \quad (40)$$

The derivative dS/dH is positive, ($H \neq 0$). On the contrary the second derivative is $d^2S/dH^2 < 0$ for $H \ll \frac{n-1}{n} (V \cdot \varepsilon \cdot \varphi^n)^{-1}$ and $d^2S/dH^2 > 0$ for $H \gg \frac{n-1}{n} (V \cdot \varepsilon \cdot \varphi^n)^{-1}$, which leads to the conclusion, that in every case there must exist a point of inflection, $d^2S/dH^2 = 0$, $H = H_w$. The shape of the characteristic curve is sigmoid as usually observed in transition processes, see Fig. 1. Mostly the density function is represented graphically with a decimal logarithmic scale of the abscissa, but we prefer the scale

$$X = \ln H/H_0 = \ln H \quad (41)$$

with the unit $H_0 = 1$. Such a representation, see Fig. 2, gives some attributes of the function and the influence of the parameters in a better way. At the characteristic curve four regions are distinguished:

1. the toe of the curve, presented by $S = S_\infty a_1 H^n = S_\infty a_1 e^{nX}$,
2. the linear part with the inflection point in its center with the maximal gradient

$$\gamma_x = \gamma_{xw},$$

3. the shoulder and
4. the saturation.

In the commonly accepted representation $S(X)$ the inflection points S_{wx} lays at higher amounts of density than in the likewise representation $S(H)$ and because of the monotony of the function at higher intensities, i.e., $\ln H_w < X_w$.

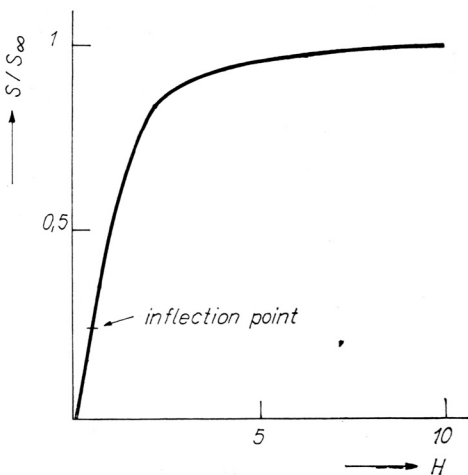


Fig. 1. Already a simple function as $S(H) = S_\infty H^2/(1 + H^2)$ shows the essential features of a characteristic curve. The inflection point is located at $H_w = \sqrt{3}/3 \approx 0.577$, $S_{wh} = 1/4$. Near 0 ($H \ll 1$) the curve shows a parabolic shape, which is not to see here.

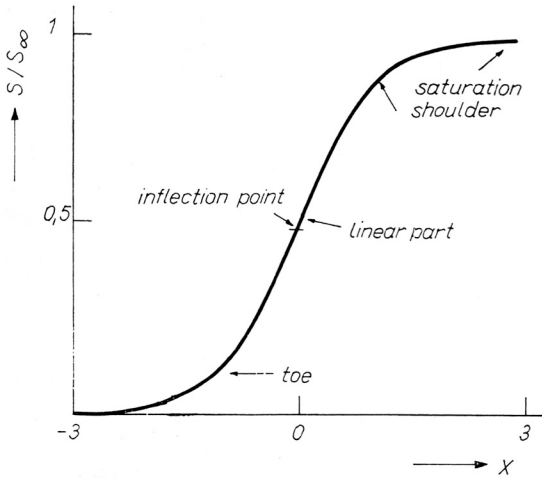


Fig. 2. The same function as in Fig. 1 but plotted over $X = \ln H$ shows the typical features of the density curve with the different regions. The inflection point is shifted to higher density and intensity values compared to Fig. 1, $X_w = 0$, (related to $H = 1$), $S_{wx} = 1/2$.

The influence of the parameters contained in the function could easily be derived by means of the above presented formulae. The saturation density S_∞ constitutes the scale of the density. The sensitivity coefficient $a = \varepsilon/\alpha_1$ effects a parallel shift of the curve in direction of the X -coordinate. On the other hand a acts on the scale of H . Apart of this, it is obvious that the reaction step number n causes either a contraction or an expansion of the curve along the abscissa, changing the slope, $dS/dX \sim n$. The grain-size scattering is characterized mainly by the ratio $\alpha = \alpha_1/\alpha_2$ with $\alpha < 1$. The influence of $\alpha = 1/16$ (a realistic value) is, compared with $\alpha = 0$, insignificant. The curve with $\alpha = 0$ lays under the curve with $\alpha \neq 0$, and the slope is a little flatter. The reason for this assertion is that for lower values α more small grains are contained in the emulsion contributing very little to the entire density. The opacity of the photolayer, denoted by the parameter $D > 0$, affects the emulsion less sensitive with increasing intensity (it is unaffected at $H = 0$).

One can say, the opacity flattens the curves. Especially for the simple case (36)

$$X_w = \frac{1}{n} \ln a + D/2, \quad \lambda = 3, \tag{42}$$

the inflection point is shifted along the X -axis by $-D/2$, but S_w maintains the same density value:

$$\frac{1}{S_\infty} \cdot S_w = \frac{1}{S_\infty} S(X_w) = \frac{1}{nD} \ln \frac{1 + e^{nD/2}}{1 + e^{-nD/2}} \equiv \frac{1}{2}, \quad \lambda = 3. \tag{43}$$

The inflection point gradient depends strongly on D ,

$$\frac{1}{S_\infty} \cdot \gamma_{Xw} = \frac{1}{S_\infty} S'(X_w) = \frac{1}{D} \cdot \frac{e^{nD/2} - 1}{e^{nD/2} + 1} \leq \frac{1}{2} \gamma_{Xw} \Big|_{D=0} = \frac{n}{4}, \tag{44}$$

and tends to zero for high values D :

$$\frac{1}{S_\infty} S'(X_w) \xrightarrow{D \rightarrow \infty} 1/D. \tag{45}$$

At last we consider the influence of the parameter λ .

Therefore we compare the equations (34) and (35). Generally, $S_2(X) > S_3(X)$. The inflection points are determined by

$$X_{w2} = \frac{1}{n} \left(\ln \frac{1}{a} - \ln \frac{3}{2} \right) < X_{w3} = \frac{1}{n} \ln a \quad (46)$$

and

$$\frac{1}{S_\infty} S_2(X_{w2}) \approx 0,536 > \frac{1}{S_\infty} S_3(X_{w3}) = 0,5,$$

$$\frac{1}{S_\infty} S_3'(X_{w3}) = \frac{n}{2} > \frac{1}{S_\infty} S_2'(X_{w2}) = \frac{n}{(5/3)^{3/2}} \approx \frac{n}{2,15} > S_2'(X_{w3}) = \frac{3\sqrt{2}}{16} n \approx \frac{n}{3,77}.$$

The characteristic curve in the case $\lambda = 2$ lays above that for $\lambda = 3$. This reflects the fact, that only the developable specks on the surface of the grain are capable to reduce the entire grain and therefore they will contribute to the density. The inflection point has a higher value for $\lambda = 2$ in comparison to $\lambda = 3$, the gradient in this point, however, is greater in the case $\lambda = 3$. In this manner the loss of density is partly compensated at higher intensities.

In this consideration a unique saturation density was presumed.

I would like to thank Dr. E. GERTH for discussions and useful clarifications of the theory.

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Received: June 12, 1987.

Author's address:

K.-O. Eschrich, Behringstr. 4, Potsdam, DDR-1590.